Scattering in Junction by Posts Consisting of a Segment of Conducting Cylinder

Rafal Lech, Michal Polewski, and Jerzy Mazur

***Abstract*—The theory of scattering in junction by posts consisting of a segment of conducting cylinder is developed using a combination of a modified iterative scattering procedure and an orthogonal expansion method. The multimode scattering matrix of the junction is obtained. The change of the posts configuration, as well as a simple rotation of the post, makes it possible to vary the electrical characteristic. The validity and accuracy of the method is verified by comparing the numerical results with those given in literature, received from finite-difference time-domain (FDTD) simulation and our own measurements. The proposed procedure is efficient and faster than alternative numerical solutions such as the FDTD method used for comparison.**

***Index Terms*—Conducting posts discontinuity, electromagnetic scattering, waveguide junction.**

I. Introduction

THE purpose of this paper is to use a combination of a modified iterative scattering procedure and an orthogonal expansion method [1]–[8] to create a fast procedure for analyzing the electromagnetic-wave scattering from two-dimensional posts in a rectangular waveguide. This problem requires distinguishing an interior area, which can be matched with other external fields. The methods proposed in [1]–[3] establish cylindrical interaction region (see Fig. 1) as an interior area on the surface of which a total scattered field from all cylinders can be found. The presented approach assumes that the cylinders are excited by unknown incident fields defined as an infinite series of Bessel functions of the first kind with unknown coefficients a_n. Their superposition determines a total scattered field from all posts on a contour R. It allows to match it with other known incident fields and to define the scattering matrix of the considered structure.

The proposed method enables us to analyze scattering by conducting posts and can be applied to research waveguide structures where incident fields are of the TE_{n0} mode. The posts consist of cylinder sector and, due to its asymmetry, it is possible to vary the electrical characteristic of such an element by means of a simple rotation. There is a possibility of realizing structures by cascading cylindrical sections. This technique was applied to model tunable filters proposed in [9].

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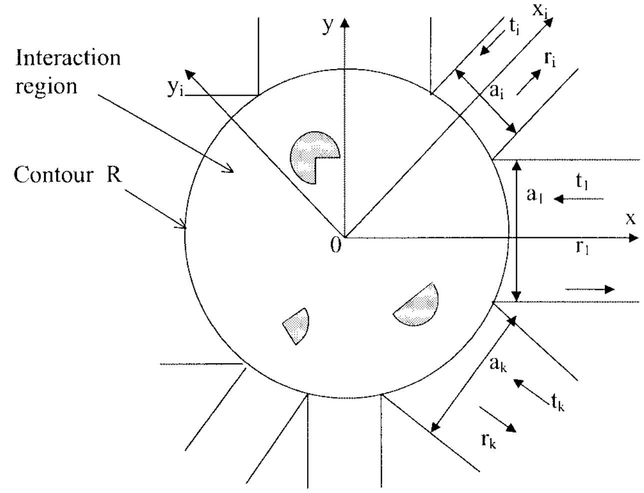

Fig. 1. Schematic representation of the scattering in a waveguide junction by conducting posts.

II. Theory

The scattered field from cylinders described on the surface of a separated interaction region can be combined with the fields of each waveguide's port (see Fig. 1). The electromagnetic field is assumed to be independent of z of the cylindrical coordinates (ρ, ϕ, z). Therefore, both scattered and waveguide fields are depicted as TE_{n0} waves and it is sufficient to consider the scattering problem with field components E_z, H_ϕ, and H_ρ.

A. Waveguide Structures

The total electric field E_z^i for the ith port described in the local Cartesian coordinate system (x_i, y_i, z) is written as follows:

$$E_z^i = \sqrt{\frac{2}{a_i b}} \sum_{n=1}^{N} \sin\left[\frac{n\pi}{a_i}\left(y_i + \frac{a_i}{2}\right)\right] \times \left(t_n^i e^{jk_{xn}^i x_i} + r_n^i e^{-jk_{xn}^i x_i}\right) \quad (1)$$

where t_n^i and r_n^i are the transmission and reflection coefficients, respectively, $k_{xn}^i = \sqrt{\omega^2 \epsilon_0 \mu_0 - (n\pi/a_i)^2}$, $i = 1, 2, \ldots, K$, and K denotes the number of waveguide ports, while the magnetic fields H_x^i and H_y^i can be determined from E_z^i.

To define the scattering matrix of the investigated structure, we transform the field components in each port i to the basic coordinate system (x, y, z) and write them in cylindrical coordinates. It allows us to enforce the continuity conditions between tangential electric and magnetic fields on a contour R of the cylindrical region and waveguide ports.

By orthogonalizing the set of eigenmode functions of the field in each waveguide port using eigenfunctions $\{e^{-jm\phi}\}$ and $\phi(0, 2\pi)$, we obtain

$$\begin{aligned}\left[E_z^T\right] &= \left[K^{ET}\right]\left[t_1^1, t_2^1, \ldots, t_1^i, t_2^i, \ldots, t_1^K, t_2^K\right]^T \\ &\quad + \left[K^{ER}\right]\left[r_1^1, r_2^1, \ldots, r_1^i, r_2^i, \ldots, r_1^K, r_2^K\right]^T \\ \left[H_\phi^T\right] &= \left[K^{HT}\right]\left[t_1^1, t_2^1, \ldots, t_1^i, t_2^i, \ldots, t_1^K, t_2^K\right]^T \\ &\quad + \left[K^{HR}\right]\left[r_1^1, r_2^1, \ldots, r_1^i, r_2^i, \ldots, r_1^K, r_2^K\right]^T \end{aligned} \quad (2)$$

where $\left[K^{ET}\right]$, $\left[K^{ER}\right]$, $\left[K^{HT}\right]$, and $\left[K^{HR}\right]$ are square matrices of the size $(2M+1)$ (M: the number of eigenfunctions in the cylindrical interaction region) described in [1].

Using the relation between the E_z and H_ϕ scattered field from cylinders on the contour R, as shown in [2], we derive the modal scattering matrix of the circuit in the following form:

$$[S] = \left(\left[K^{ER}\right] - [Z]\left[K^{HR}\right]\right)^{-1}\left([Z]\left[K^{HT}\right] - \left[K^{ET}\right]\right). \quad (3)$$

B. Interaction Region

In order to find matrix $[Z]$, defined in (3), we use a combination of a modified iterative scattering procedure and the orthogonal expansion method described in [1] for cylindrical obstacles. In this approach, we consider harmonic E_z-wave excitation as an infinite series of Bessel functions of the first kind with unknown coefficients a_m

$$E_z^{\text{inc}(0)} = \sum_{m=-\infty}^{\infty} a_m J_m(k_0\rho)e^{jm\phi}. \quad (4)$$

This field excites a number of conducting posts and has to be defined in their local coordinates. For the ith post, we obtain

$$E_{zi}^{\text{inc}(0)} = \sum_{m=-\infty}^{\infty} a_m \sum_{p=-\infty}^{\infty} J_p(k_0 r_i)e^{jp\phi_i} \cdot J_{p-m}(k_0 d_{io})e^{j(m-p)\phi_{io}} \quad (5)$$

where d_{io} and ϕ_{io} are the distance and angle between the center of the local and global coordinate system.

The scattered and transmitted field components for the ith post can also be described as a series of Bessel functions with unknown coefficients b_m and A_n, respectively. For the investigated structure, shown in Fig. 1, these fields can be expressed as

$$E_{zi}^{s(0)} = \sum_{m=-\infty}^{\infty} b_{im} H_m^{(2)}(k_0\rho)e^{jm\phi} \quad (6)$$

$$E_{zi}^{t(0)}(\rho, \phi_i) = \sum_{n=1}^{\infty} A_{\text{in}} J_l(k_i\rho_i)\sin\left(\frac{n\pi(\phi_i - \phi_{0i})}{2(\pi - \theta_i)}\right) \quad (7)$$

where $k_i = k_0\sqrt{\epsilon_r}$, $l = n\pi/2(\pi - \theta)$, J_l, and $H_m^{(2)}$ are Bessel and Hankel functions.

Forcing the continuity of the tangential components of electric and magnetic fields on the surface of each contour ξ and using the orthogonalization method, we obtain the unknown coefficients b_m. Taking into account $m = 2M + 1$, the solution is expressed as

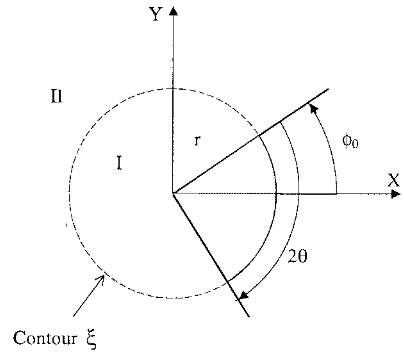

Fig. 2. Cross section of single post and its subregions for analysis.

$$[b] = [G]\left[T_{io}\right][a] \quad (8)$$

where $[b] = \left[b_{-M}, \ldots, b_0, \ldots, b_M\right]^T$, $[a] = \left[a_{-M}, \ldots, a_0, \ldots, a_M\right]^T$, and $[G]$ is a matrix describing the relation between unknown coefficients b_m and a_m, and $[T_{io}]$ is the transformation matrix of Bessel functions from global coordinates to the local coordinates of the ith cylinder [1].

Applying the iterative scattering procedure [1], [4], we use the scattered field from previous iteration as a new incident field on the remaining posts. The coefficients of the pth iteration depend only on the coefficients of the $(p-1)$th iteration. Using this method, we determine the total electric and magnetic fields on the surface of the interaction region. Hence, the relation between electric and magnetic fields on the surface of a cylindrical interaction region is written as

$$\left[E_z^T\right] = [Z]\left[H_\phi^T\right] \quad (9)$$

where $[Z]$ is the demanded matrix.

C. Scattering Field of a Single Post

The transmitted field described as follows by (10) gives the axial component of the electric field E_z^{I} in Region I (see Fig. 2):

$$E_z^{\text{I}}(\rho, \phi) = \sum_{n=1}^{\infty} A_n J_l(k_i\rho)\sin\left(\frac{n\pi(\phi - \phi_0)}{2(\pi - \theta)}\right). \quad (10)$$

The incident (5) and scattered (6) fields in Region II are written as follows:

$$E_z^{\text{II}}(\rho, \phi) = \sum_{m=-\infty}^{\infty}\left(a_m J_m(k_i\rho) + b_m H_m^{(2)}(k_i\rho)\right)e^{jm\phi} \quad (11)$$

where J_m and $H_m^{(2)}$ are Bessel and Hankel functions of order m, a_m, and b_m are unknown coefficients.

Applying the continuity condition at interface I–II $(\rho = r)$ and assuming the tangential component of electric field E_z at the interface I–II by an unknown function $U(\phi)$, we obtain

$$U(\phi) = \begin{cases} 0, & \phi_0 + 2(\pi - \theta) \leq \phi \leq \phi_0 \\ E_z^{\text{I}}(\rho = r, \phi) = E_z^{\text{II}}(\rho = r, \phi), & \phi_0 \leq \phi \leq \phi_0 + 2(\pi - \theta) \end{cases} \quad (12)$$

and

$$H_\phi^{\mathrm{I}}(\rho=r,\phi)-H_\phi^{\mathrm{II}}(\rho=r,\phi)=J_z,\qquad \phi_0+2(\pi-\theta)\le\phi\le\phi_0. \tag{13}$$

If we apply the boundary conditions (12) to the field (10) and (11) and orthogonalize (10) by $\{\sin(n\pi(\phi-\phi_0)/2(\pi-\theta))\}$ and (11) by $\{e^{-jm\phi}\}$, we obtain the unknown coefficients A_n and a_m as a function of b_m

$$A_n=\frac{1}{J_l(k_ir)}\int_{\phi_0}^{\phi_0+2(\pi-\theta)}U(\phi)\sin\left(\frac{n\pi(\phi-\phi_0)}{2(\pi-\theta)}\right)d\phi \tag{14}$$

$$a_m=\frac{1}{J_m(k_ir)}\left(\int_{\phi_0}^{\phi_0+2(\pi-\theta)}U(\phi)e^{-jm\phi}d\phi-b_mH_m^{(2)}(k_ir)\right). \tag{15}$$

We now expand function $U(\phi)$ in a series of basis functions with unknown coefficients c_k and assume the number of terms in expansion K as follows:

$$U(\phi)=\sum_{k=1}^{K}c_kU_k(\phi). \tag{16}$$

The basis functions should contain as much information we have on the behavior of the tangential electric field at the interface as possible; its conditions at the sharp metallic edges of the ridge especially. A set of basis functions satisfying this local requirement is undertaken from [10] and is given as follows:

$$U_k=\frac{\sin\left[\dfrac{k\pi(\phi-\phi_0)}{2(\pi-\theta)}\right]}{\sqrt[3]{(\phi-\phi_0)\left[2(\pi-\theta)-(\phi-\phi_0)\right]}}. \tag{17}$$

Applying (16) and (17) to (14) and (15), we receive

$$A_n=\frac{1}{J_l(k_ir)}\sum_{k=1}^{K}c_kU_{kn}^0 \tag{18}$$

$$a_m=\frac{1}{J_m(k_ir)}\left(\sum_{k=1}^{K}c_kU_{km}^\phi-b_mH_m^{(2)}(k_ir)\right). \tag{19}$$

Forcing the continuity of the tangential magnetic field at the interface (13) and orthogonalize it using base functions as weight functions, we obtain

$$\sum_{n=1}^{\infty}A_nJ_l'(k_ir)U_{jn}^0-\sum_{m=-\infty}^{\infty}\left(a_mJ_m'(k_ir)+b_mH_m^{(2)'}(k_ir)\right)\times U_{jm}^{\phi*}=0,\qquad \text{for } j=1,\ldots,K. \tag{20}$$

The following notations were introduced for convenience:

$$U_{kn}^0=\int_0^{2(\pi-\theta)}\frac{\sin\left[\dfrac{k\pi(\phi')}{2(\pi-\theta)}\right]}{\sqrt[3]{(\phi')\left[2(\pi-\theta)-(\phi')\right]}}\sin\left(\frac{n\pi\phi'}{2(\pi-\theta)}\right)d\phi' \tag{21}$$

$$U_{km}^\phi=e^{-jm\phi_0}\int_0^{2(\pi-\theta)}\frac{\sin\left[\dfrac{k\pi(\phi')}{2(\pi-\theta)}\right]}{\sqrt[3]{(\phi')\left[2(\pi-\theta)-(\phi')\right]}}e^{-jm\phi'}d\phi' \tag{22}$$

where $\phi'=\phi-\phi_0$.

Applying (18) and (19) into (20), we determine the coefficients c_k

$$[c]=[D]^{-1}[R][b]=[P][b] \tag{23}$$

where $[c]=[c_1\ \ldots\ c_K]^T$, $[b]=[b_{-M}\ \ldots\ b_{-1}\ b_0\ b_1\ \ldots\ b_M]^T$

$$[D]=\left[\left(\sum_{n=1}^{N}\frac{J_l'(k_ir)}{J_l(k_ir)}U_{kn}^0U_{jn}^0\right)-\left(\sum_{m=-M}^{M}\frac{J_m'(k_ir)}{J_m(k_ir)}U_{km}^\phi U_{jm}^{\phi*}\right)\right]_{k=1,j=1}^{K,K}$$

$$[R]=\left[-\left[\frac{H_m^{(2)}(k_ir)J_m'(k_ir)}{J_m(k_ir)}-\frac{J_m(k_ir)H_m'^{(2)}(k_ir)}{J_m(k_ir)}\right]U_{jm}^{\phi*}\right]_{j=1,m=-M}^{K,M}.$$

Substituting (23) for (19), after some algebra, we obtain the demanded relation between unknown coefficients a_m and b_m

$$[b]=[F]^{-1}[a]=[G][a] \tag{24}$$

where $[a]=[a_{-M}\ \ldots\ a_{-1}\ a_0\ a_1\ \ldots\ a_M]^T$ and

$$[F]_{pq}=\begin{cases}\dfrac{1}{J_p(k_ir)}\sum_{k=1}^{K}U_{k,p}^\phi P_{k,p}-\dfrac{H_p^{(2)}(k_ir)}{J_p(k_ir)}, & \text{for } p=q\\ \dfrac{1}{J_p(k_ir)}\sum_{k=1}^{K}U_{k,p}^\phi P_{k,q}, & \text{for } p\ne q\end{cases} \tag{25}$$

for $p,q=-M,\ldots,-1,0,1,\ldots,M$.

III. Experiment

For numerical investigation, the infinite sums used in the mathematical formulations were replaced by finite sums. From the simulations, we obtain a good convergence for a number of eigenfunctions $M=10$ and basis functions $K=7$. This effect is depicted in Table I, where the convergence of $|S_{11}|$ calculations is presented. With the chosen numbers M and K, the computation of one scattering matrix for a single-post configuration needs approximately 0.52 s on a MATLAB, 850-MHz Intel Celeron personal computer (PC).

There is a possibility of using this method for filters design by cascading single cylindrical sections. In order to test the validity of the method, the filter structure proposed in [9] was investigated. The performance of the filter is predicted by cascading S-matrices of the separated sections. The schematic representation of the structure consisting of four half-cylinder inductive

TABLE I

COMPARISON OF THE PERCENTAGE ERROR $\delta^{m,k}[\%] = (|S_{11}|^{M,K} - |S_{11}|^{m,k} / |S_{11}|^{M,K})$ FOR THE CONVERGENCE OF $|S_{11}|$ FOR A DIFFERENT NUMBER OF HARMONICS M VERSUS THE NUMBER OF BASIS FUNCTIONS K FOR A SINGLE-POST CONFIGURATION: $r = 8$ mm, $d = 3$ mm, $\phi = 0°$, $\theta = 90°$ $f = 11$ GHz

		$k = 3$	4	5	6	7
$m = 6$	$\delta[\%]$	2.454	0.151	0.500	0.182	0.364
7	$\delta[\%]$	2.439	0.364	0.757	0.606	0.621
8	$\delta[\%]$	1.788	0.227	0.273	0.151	0.167
9	$\delta[\%]$	1.742	0.197	0.348	0.258	0.273
10	$\delta[\%]$	1.348	0.545	0.045	0.030	0.000

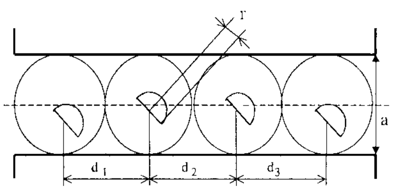

Fig. 3. Schematic representation of a filter consisting of half-cylinder inductive posts. Their central points are displaced by 5.5, 2.2, 2.2, and 5.5 mm with respect to the waveguide axis. The distances between the post are $d_1 = 21.18$ mm, $d_2 = 23.09$ mm, and $d_3 = 22.20$ mm.

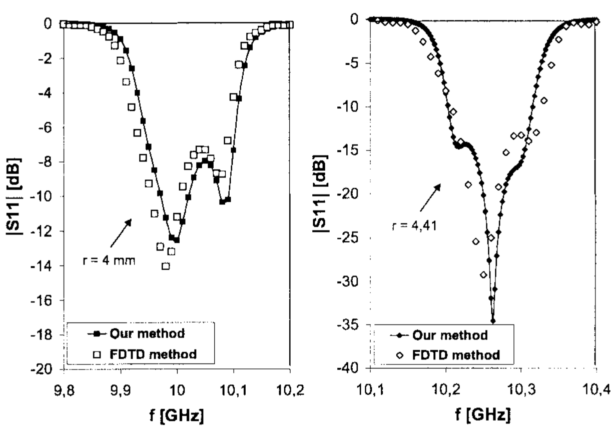

Fig. 4. Frequency responses of a filter from Fig. 3 for $r = 4$ mm and $r = 4.41$ mm.

posts, and the results are presented in Figs. 3 and 4. The results show that by using our method, a good agreement with calculations of the commercial finite-difference time-domain (FDTD) simulator *Quick-Wave 3D*[1] was obtained. The possibility of applying our method to the investigation of other structures follows from the results. Due to not clearly specified parameters of the filter in [9], we found it difficult to compare our results with those given in [9].

The performance of single- and double-post configurations were simulated and compared with the numerical results derived from *Quick-Wave 3D* and our measurements. The experiment was performed using the Wiltron 37269A Network Analyzer. Figs. 5–8 show the calculated and measured frequency responses of the reflection coefficient $|S_{11}|$ for several post radii r, values of θ angle, and positions d. Those configurations can be used as tuned bandpass filters.

[1][Online]. Available: http://www.qwed.com.pl

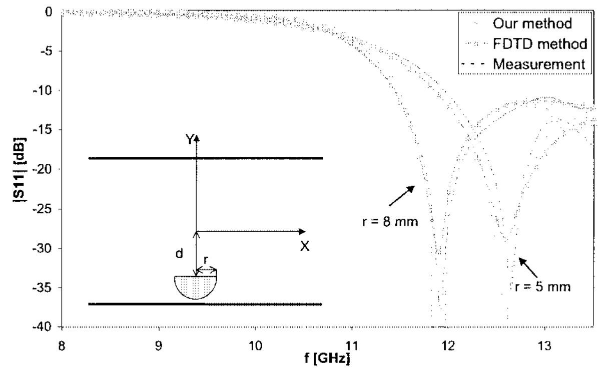

Fig. 5. Frequency responses of a single-post configurations: $r = 5$ mm, 8 mm, $d = 3$ mm, and $\phi_0 = 0°$.

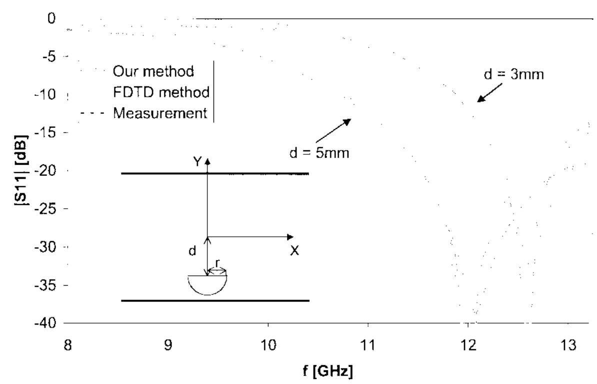

Fig. 6. Frequency responses of a single-post configurations: $r = 5$ mm, $d = 3$ mm, 5 mm, and $\phi_0 = 0°$.

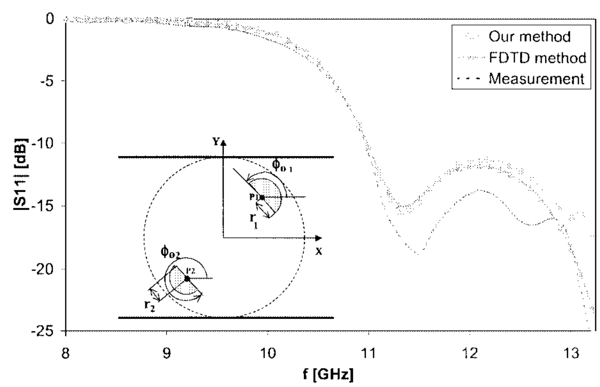

Fig. 7. Frequency response of a double-post configurations: $r_1 = 4$ mm, $\phi_1 = 150°$, $r_2 = 3$ mm, and $\phi_2 = 310°$. Coordinates of the central points of the posts are $P1 = (5 \text{ mm}, 5 \text{ mm})$ and $P2 = (-5 \text{ mm}, -5 \text{ mm})$.

When the radius r of the post for the constant displacement d is increased, the resonance frequency decreases (see Fig. 5). The same situation is seen in Fig. 6 while moving the post of constant radius r from the waveguide center to one side of the wall. Thus, a certain resonance frequency can be achieved by two alternative post positions. The calculated effect was proven by measurement. The measured attenuation is smaller for the single-post structures than the calculated value because of the waveguide and measurement device losses, which are not considered in the analysis. The discrepancies that occurred in double-post configuration (see Fig. 7) are mainly caused by high sensitivity of the investigated configuration for displacement, which has a great

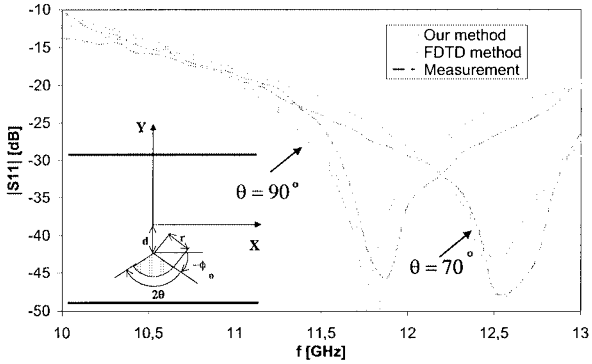

Fig. 8. Frequency response of a single-post configurations: $r = 5$ mm, $d = 6.4$ mm, $(\theta = 90°, \phi = 0°)$, and $(\theta = 70°, \phi = -20°)$.

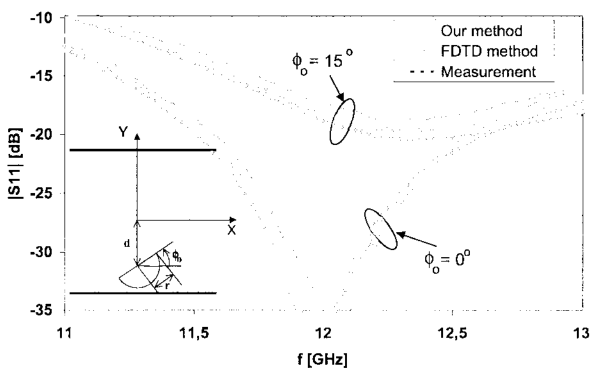

Fig. 9. Frequency response of a single-post configuration. $r = 5$ mm, $d = 5$ mm, and $\phi_0 = 0°$, and 15°.

influence on the resonance frequencies. Fig. 8 shows the change of the resonance frequency for different values of the θ angle.

The results of numerical analysis and the measurements show (see Fig. 9) that rotation of the post causes the variation of the resonance frequency. In cascade structures, the rotations relate to the variation of the effective cavity lengths, thus permitting some tuning. Therefore, once the filter structure is realized, it is possible to adjust its frequency response by slightly rotating the posts.

IV. Conclusion

The analysis for scattering in junction by posts consisting of a segment of conducting cylinder has been developed using a combination of a modified iterative scattering procedure and an orthogonal expansion method. This approach is convenient for investigating the waveguide structures. Since this procedure works much faster than *Quick-Wave 3D*, it is suitable for the synthesis of waveguide filters. The inclusion of the edge conditions in the basis functions at the sharp metallic edges of the ridges makes the approach numerically efficient. The tuning of the resonance frequencies has been observed with the rotation, segmentation, changing radius, and off-axis displacement of the metallic posts located in a rectangular waveguide.

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